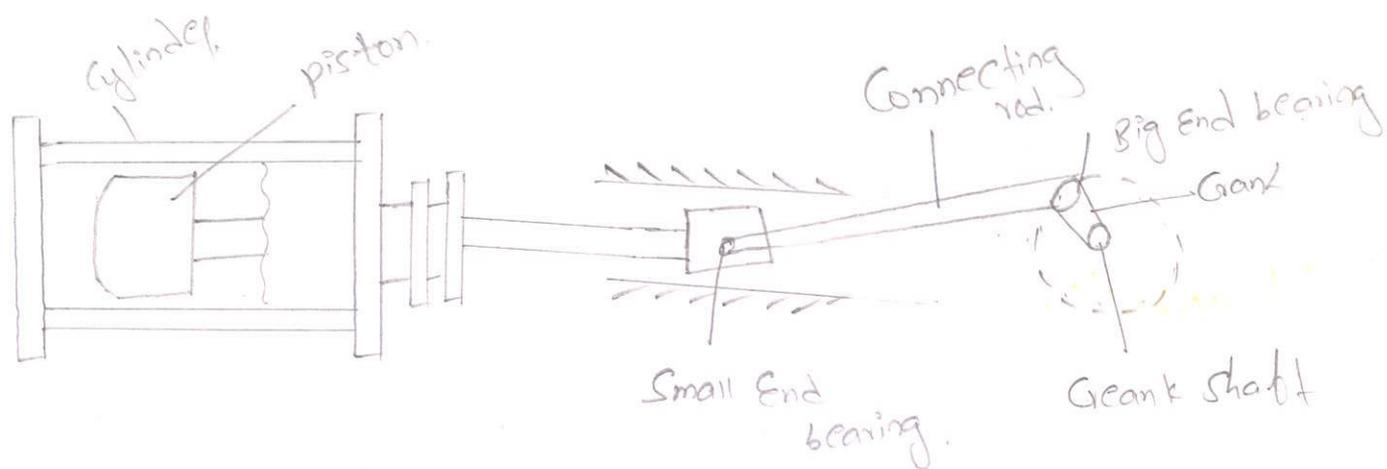


# UNIT-1

Introduction :- A mechanism (or) machine is a device which receives energy and transforms it into some useful work. A machine consists of number of parts or bodies. In this chapter, we shall study the mechanisms of the various parts or bodies from which the machine is assembled. This is done by making one of the parts as fixed, and the relative motion of other parts is determined with respect to the fixed part.

Kinematic Link or Element :-

Each part of the machine which moves relative to some other part is known as a "Kinematic link" (or simply link) or element. A link may consist of several parts, which are rigidly fastened together so that they do not move relative to one another.



Thus a link should have the following

two characteristics.

1. It should have relative motion, and.
2. It must be a resistant body.

Types of links:-

Rigid link:- A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of reciprocating stem engine is not appreciable,

Fluid link:- A fluid flexible link is one which is partly deformed in a manner not to affect the motion is transmitted through the fluid by.

Theory of machine is that branch of science which deals with study of relative motion b/w the various parts of machine, and forces which act on them. Theory of machine may be divided into kinematics and dynamics.

Kinematics :- (i) This deals with the relative motion b/w the various parts of the machine.

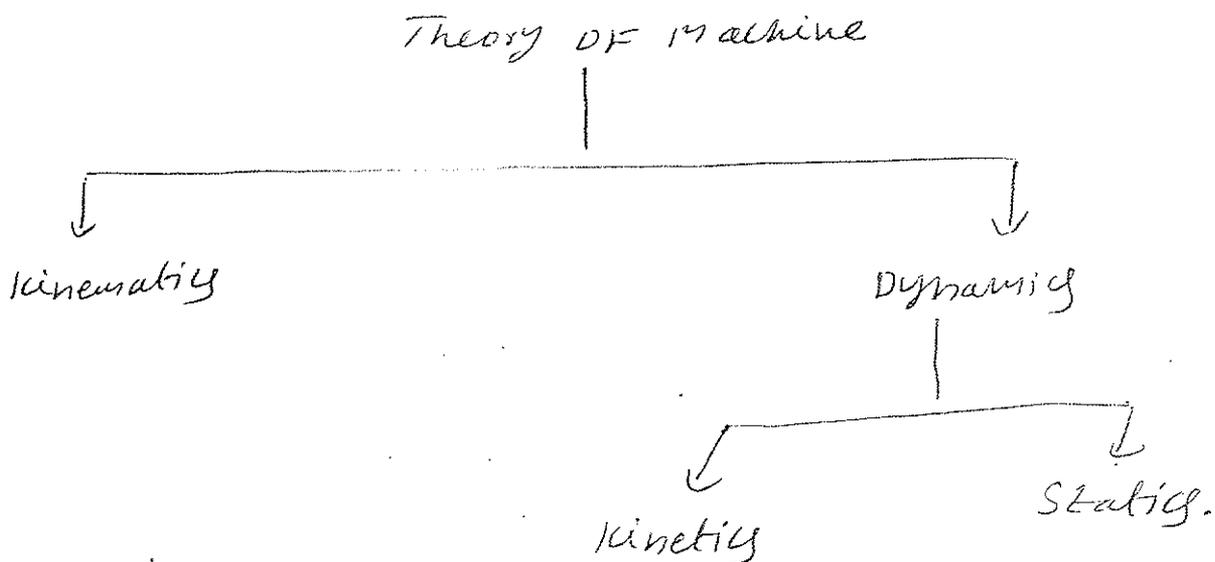
(ii) various forces involved in the motion are not considered.

Dynamics :- (i) This deals with the study of various forces involved in the various parts of the machine.

(ii) Forces may be static or dynamic. This are two types

a) Kinetics :- This deals with the various forces when body is moving.

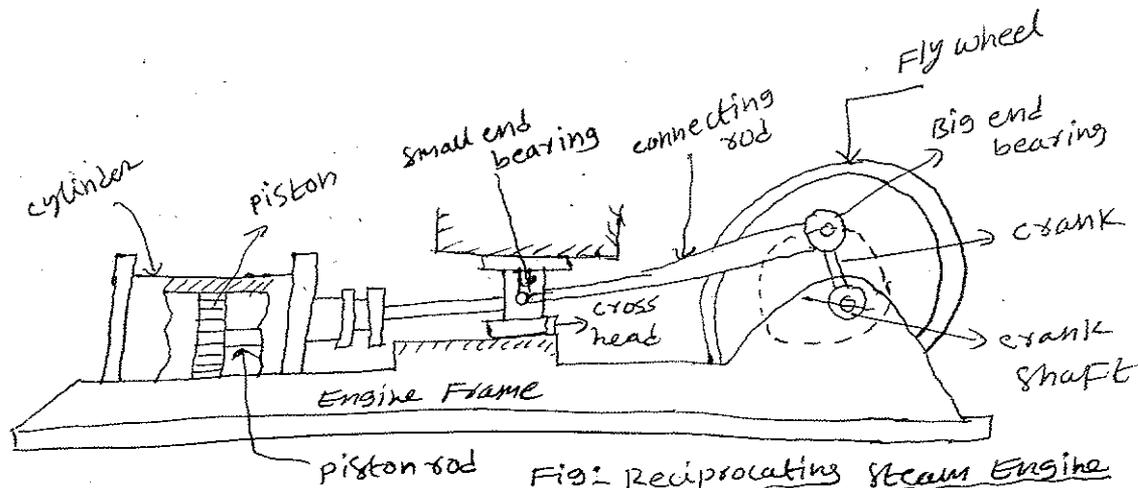
(b) Statics :- This deals with the various forces when body is stationary.





## ⊕ Kinematic Link or Element :-

Each part of a machine, which moves relative to some other part, is known as kinematic link or element. A link may be consists of several parts, which are rigidly fastened together, so that they do not move relative to one another.



For example, in a reciprocating steam engine, as shown in above Fig. Piston, piston rod and crosshead constitute one link. connecting rod with big and small end bearings constitute a second link. crank, crank shaft and fly wheel a third link and the cylinder, engine frame and main bearings a fourth link.

A link or element need not to be a rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics

1. It should have relative motion and
2. It must be a resistant body.

## \* Types of Links :-

In order to transmit motion, the driver and the follower may be connected by the following three types of links.

1. Rigid Link :- A rigid link is one which does not undergo any deformation while transmitting motion. strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc., of a reciprocating steam engine is not appreciable, they can be considered as rigid link.

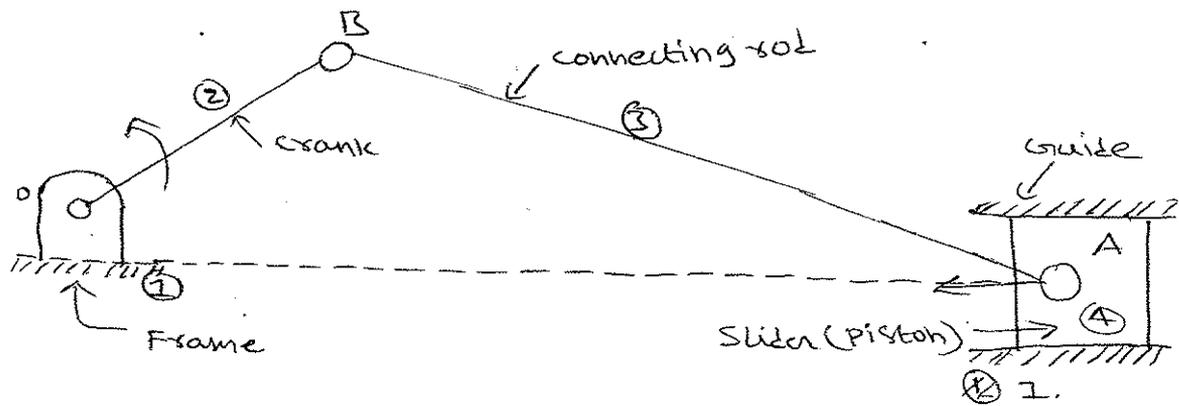
2. Flexible Link :- A flexible link is one which is partly deformed in a manner not to affect the motion is transmitted through the transmission of motion. For example, belts, ropes, chains and wires are flexible link and transmit tensile forces only.

3. Fluid Link :- A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

⊕ LINK - A link is defined as a member or a combination of members, connecting other members and having motion relative to them. A slider-crank mechanism consists of following four links as shown following Fig.

(i) Frame

(ii) crank (iii) connecting rod (iv) slider



The slider (ie link 4) reciprocates in guide, which is connected to frame. Hence guide also becomes link 1 (ie Frame)

⊕ Types of Link :- In order to transmit motion, the driver and the follower may be connected by the following three types of links.

① Rigid link :- A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc., of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

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③ Fluid Link :- A Fluid Link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, Jack and brakes.

\* Kinematic pair :-

A Joint of two links having relative motion b/w them is known as a kinematic pair. In a slider crank mechanism shown above Fig. Link 2 rotates to link 1 and hence link 1 and 2 is a kinematic pair. Similarly link 2 is having motion relative to link 3 and hence link 2 and 3 is also a kinematic pair. Link 3 is having motion relative to link 4. Also link 4 is having motion relative to link 1. Hence link 3, 4 and 4, 1 constitute kinematic pairs.

CLASSIFICATION OF KINEMATIC PAIRS :-

Kinematic pairs can be classified

(i) According to nature of contact b/w the links :- The kinematic pairs are classified as

a) Lower pair (b) Higher pair.

a) A kinematic pair is known as lower pair if the two links has surface contact or area contact between them. Also the contact surfaces of the two links are similar. Examples for lower pairs are (i) shaft rotating in a bearing (ii) nut turning on a screw.

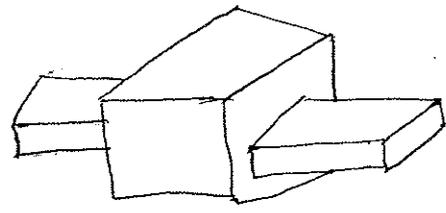
(b) IF the two links has a point or line contact between them, then the kinematic pair is known as higher pair. The contact surfaces of the two links are not similar. Example for higher pairs are (i) cam and follower and wheel rolling on a surface.

(ii) According to the type of relative motion between the two link :-

The kinematic pairs are classified as

- a) Sliding pair
- b) Turning pair
- c) Rolling pair
- d) screw pair [Helical pair]
- e) Spherical pair.

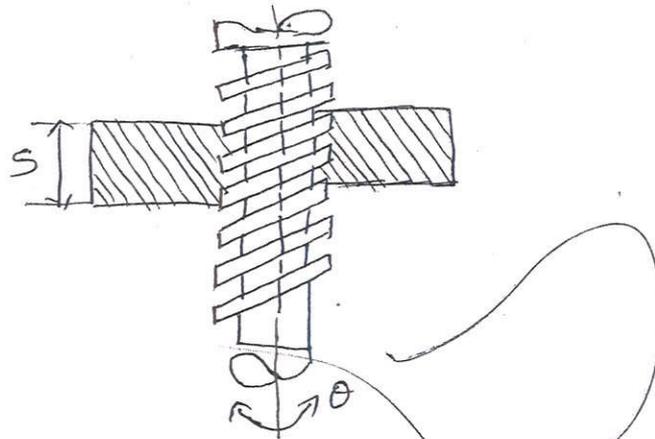
a) Sliding pair :- A kinematic pair is known as sliding pair if the two links have a sliding motion relative to each other. In Fig reciprocating steam engine, the links 4 and 1 are having sliding motion relative to each other and hence they form a sliding pair. Another example is a rectangular rod in a rectangular hole as shown in Fig below.



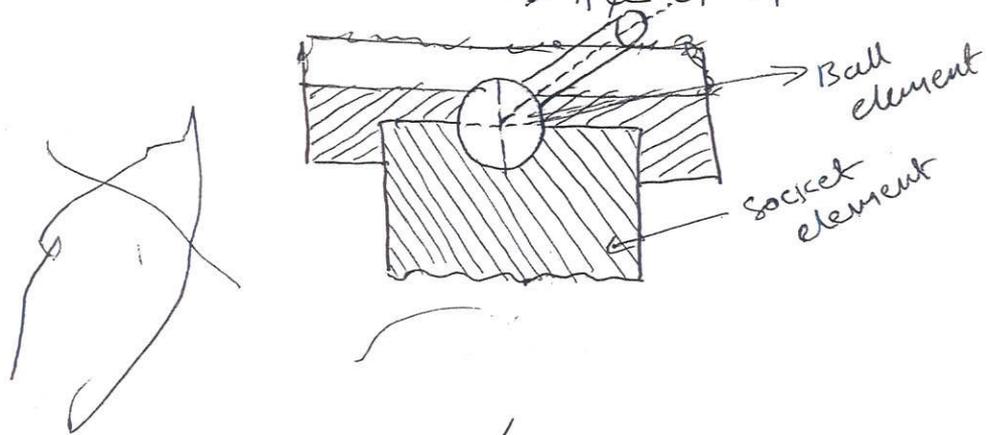
b) Turning pair :- A kinematic pair is known as turning pair if one link has a turning or revolving motion relative to the other. In Fig reciprocating link 2 is having revolving motion relative to link 1 hence link 2 and 1 constitutes a turning pair. Also the link 3 is having turning motion relative to link 4 and hence link 3 and 4 forms a turning pair. Similarly links 2 and 3 also forms turning pairs.

c) Rolling pair :- A kinematic pair is known as rolling pair if the <sup>one</sup> ~~base~~ link ~~has~~ has a rolling motion relative to the other. A rolling wheel on a flat surface forms a rolling pair. In a ball bearing, the ball and bearing forms one rolling pair whereas the ball and shaft forms second rolling pair.

(d) Screw pair :- A kinematic pair is known as screw pair if the two links have a turning as well as sliding motion between them. The Lead screw and the nut of a lathe is a screw pair. Fig Following shows another screw pair. Also bolt with a nut is another example of screw pair. If bolt is kept fixed, nut will have sliding as well as rotational motion.



(e) Spherical pair :- A kinematic pair is known as spherical pair if one link in the form of a sphere being inside a fixed link. The ball and socket joint as shown in following fig. is a spherical pair. The pen stand is another example of spherical pair.



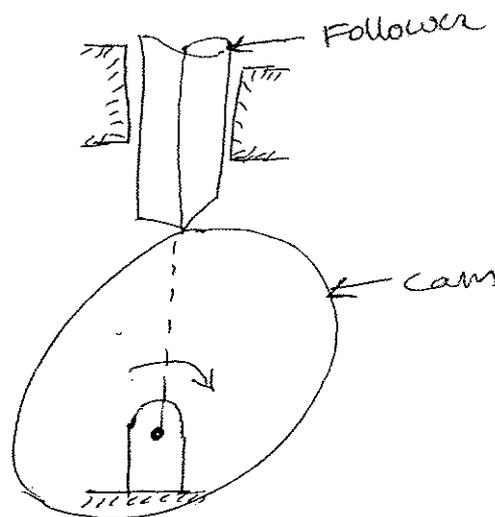
111 According to nature of mechanical constraint b/w two links :-

The kinematic pairs are classified as

(a) closed pairs and

(b) undclosed (or) open pair.

(a) closed pair :- In case of closed pair, the two elements of the pair are held together mechanically whereas in case of undclosed pair, the elements of the pair are in contact due to force of gravity or due to some spring action.



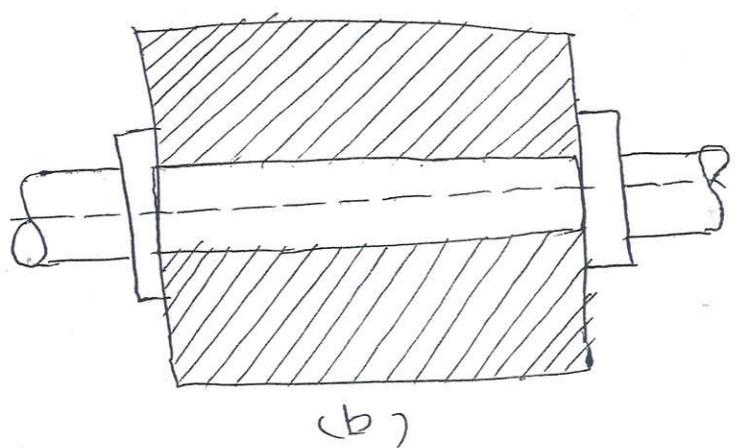
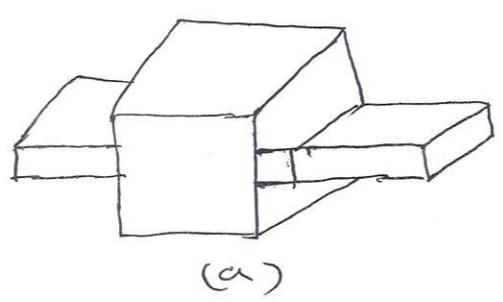
The elements are not held together mechanically for example pair of cam and follower as shown in fig. is an example of undclosed pair as it is kept in contact by the forces exerted by spring and gravity.

\* TYPES OF constrained motions \*

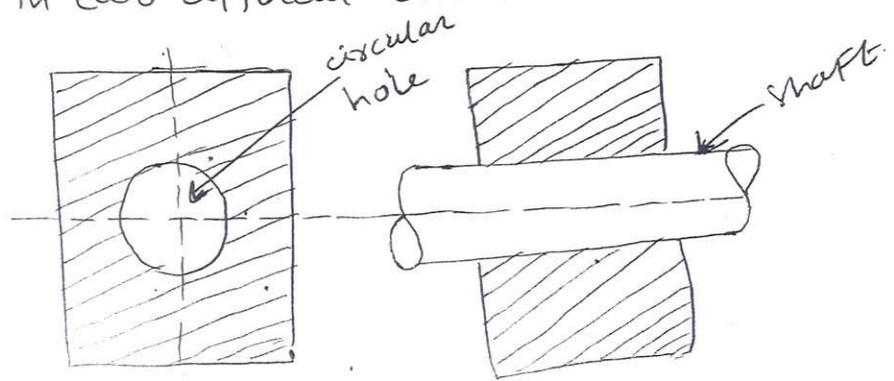
The constrained motions are of three types, namely.

- (1) completely constrained motion.
- (2) In completely constrained motion.
- (3) successfully constrained motion.

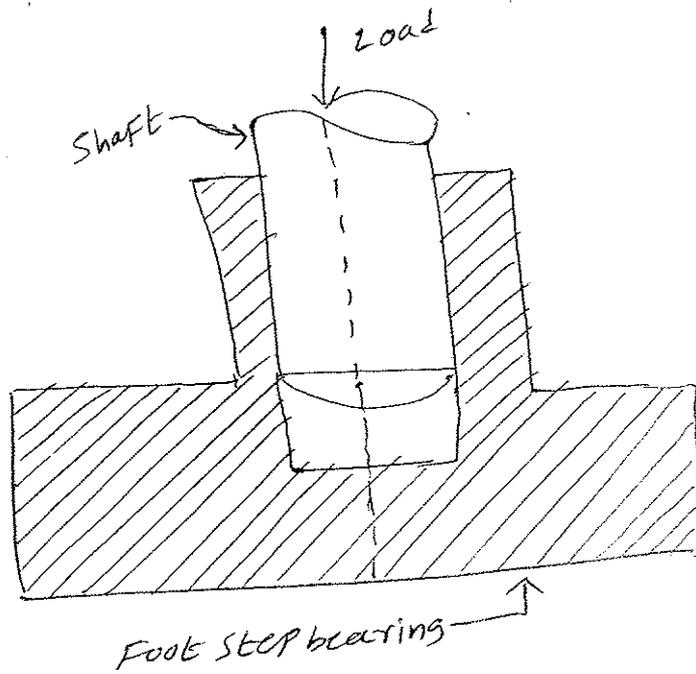
(1) completely constrained motion :- The motion b/w a pair in a definite direction irrespective of the direction of force applied, is known as a completely constrained motion. For example, The motion of a square bar in a square hole as shown in Fig. and the motion of the shaft with collar at each end in a circular hole as shown in Fig, are in a definite direction. Hence these are the example of completely constrained motion.

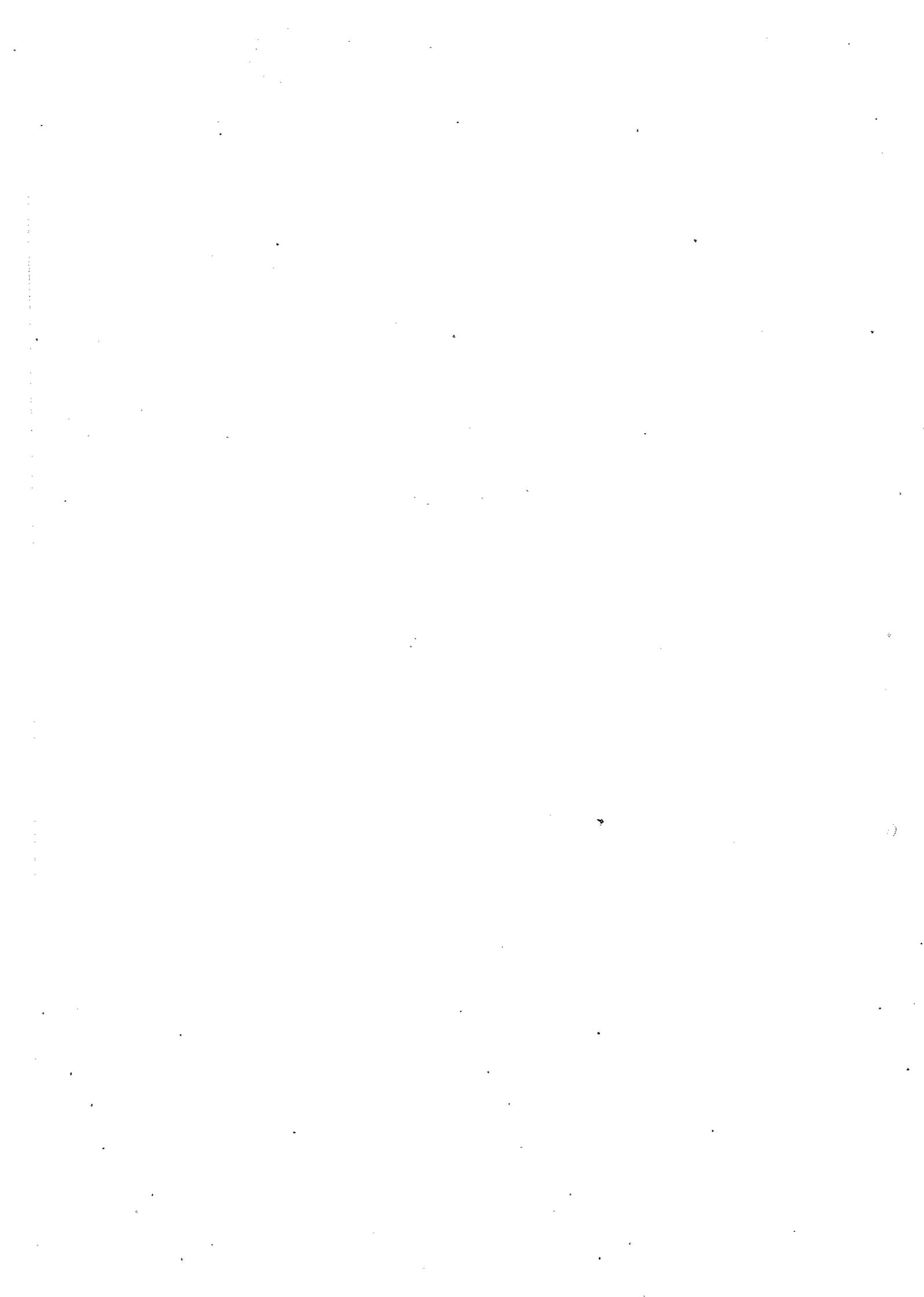


(2) In completely constrained motion :- The motion b/w a pair is known as an incompletely constrained motion if the motion b/w the pair can take place in more than one direction. The motion of a circular shaft in a circular hole as shown in Fig. is an example of incompletely constrained motion. The shaft is having motions in two different directions. It may rotate or slide in the hole.



③ Successfully constrained motion :- The motion b/w a pair is known as a successfully constrained motion if the motion b/w the pair is not completely constrained by itself, but some other means. Fig below, shows a Foot Step bearing with a shaft. The shaft may rotate in the bearing and also may move upwards. This motion is incompletely constrained. But if a load is placed on the shaft, then shaft cannot have upward movement. Then the motion of the pair becomes completely constrained. This completely constrained motion is obtained by some other means i.e by placing load on the shaft. This type of motion is known as successfully constrained motion.





\* Kinematic chain :- when the kinematic pairs are coupled in such a way that the last link coupled is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion). it is called a kinematic chain.

For example, the crank shaft of an engine forms a kinematic pair with the bearing which are fixed in a pair, the connecting rod with crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms a fourth pair. The total combination of these links is a kinematic chain.

The relation between the number of pairs ( $P$ ) and number of links [ $L$ ] in four link kinematic chain is given by

$$L = 2P - 4 \text{ ----- (1)}$$

The relation between the number of links [ $L$ ] and number of joints ( $J$ ) forming a four link kinematic chain is given by

$$J = \frac{3}{2}L - 2 \text{ ----- (2)}$$

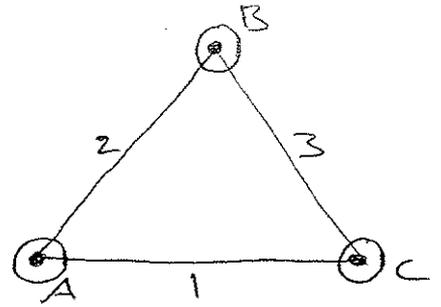
In equations (1) and (2)

$L.H.S. > R.H.S.$  Then chain is locked

$L.H.S. = R.H.S.$  Then chain is constrained

$L.H.S. < R.H.S.$  Then chain is unconstrained.

① A three links chain with three joints is shown in Fig. prove that the chain is locked.



Sol:- Given:

Three links 1, 2 and 3

Three joints A, B and C.

∴ number of joints,  $J = 3$

number of links,  $L = 3$

number of pairs,  $P = 3$

using eq ①, we get

$$L = 2P - J$$

$$3 = 2(3) - 3$$

$$3 = 2(0) \quad L.H.S > R.H.S$$

now using eq (2),

$$J = \frac{3}{2}L - 2$$

$$3 = \frac{3}{2} \times 3 - 2$$

$$3 = 2.5$$

$$L.H.S > R.H.S$$

From the above value  $L.H.S > R.H.S$ , Hence it is not a kinematic chain. Hence no relative motion is possible and the chain is known as locked chain

② A Four link chain with four joints is shown in Fig, prove that it is a constrained kinematic chain.

Sol: Given:-

number of links,  $L = 4$

number of joints,  $J = 4$

number of pairs,  $P = 4$

using the eqn,

$$L = 2P - 4$$

$$4 = 2(4) - 4$$

$$4 = 4$$

$$\therefore L.H.S = R.H.S$$

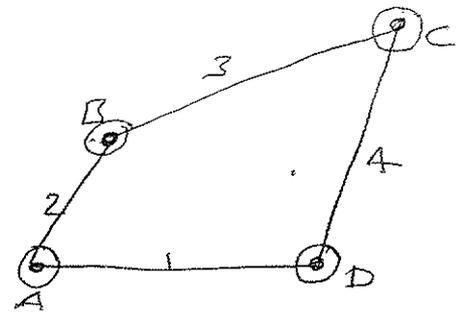
now using the eqn

$$J = \frac{3}{2}L - 2$$

$$4 = \frac{3}{2}(4) - 2$$

$$4 = 4$$

$$\therefore L.H.S = R.H.S$$



By using the above values  $L.H.S = R.H.S$ , hence the arrangement of four links as shown in above Fig, is a constrained kinematic chain.

③ Consider an arrangement of five links, as shown in Fig. prove that it is a unconstrained kinematic chain.

$L = 5$ ,  $J = 5$ , and  $P = 5$

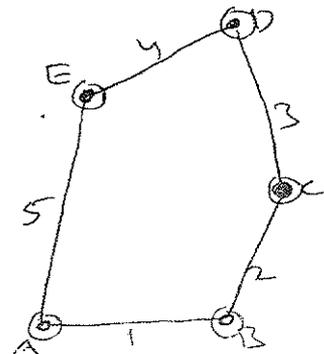
using the eqn.

$$L = 2P - 4$$

$$5 = 2(5) - 4$$

$$5 = 6$$

$$\therefore L.H.S < R.H.S$$



From the eqn

$$S = \frac{3}{2}L - 2$$

$$S = \frac{3}{2} \times 5 - 2$$

$$S = 5.5$$

$$\therefore L.H.S < R.H.S$$

From the above values  $L.H.S < R.H.S$ , Hence the arrangement of five link as shown in above fig. is unconstrained kinematic chain.

## \* Mechanism and machines \*

When one of the links of a kinematic chain is fixed, the chain is known as mechanism.

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.

The difference between machine and <sup>structure</sup> mechanisms are

Machine	Structure
1) The parts of a machine move relative to one another.	① The members of a structure do not move relative to one another.
2. A machine transforms the available energy into some useful work.	2) A structure does not transform any energy into useful work.
③ The links of a machine may transmit both power and motion	③ The members of a structure transmit forces only.

## \* Grubler's criteria

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting  $n=1$  and  $h=0$  in Kutzbach equation, we have,

$$n = 3(l-1) - 2J - h$$

$$1 = 3(l-1) - 2J - 0$$

$$1 = 3l - 3 - 2J$$

$$\Rightarrow 3l - 2J - 4 = 0$$

This eqn is known as the Grubler's criterion for plane mechanisms with constrained motion.

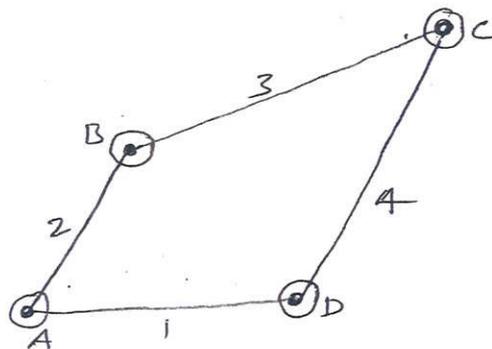
A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints can not have odd number of links. The simplest possible mechanisms of this type are a four bar mechanism and a slider-crank mechanism in which  $l=4$  and  $J=4$ .

## \* DIFFERENT TYPES OF KINEMATIC CHAINS \* AND THEIR INVERSIONS :-

The simplest kinematic chain is a chain consisting of four kinematic pairs, each pair being a sliding pair or a turning pair. The important types of kinematic chain with four kinematic pairs, are:

- (i) Four bar chain.
- (ii) Single slider crank chain, and
- (iii) Double slider crank chain.

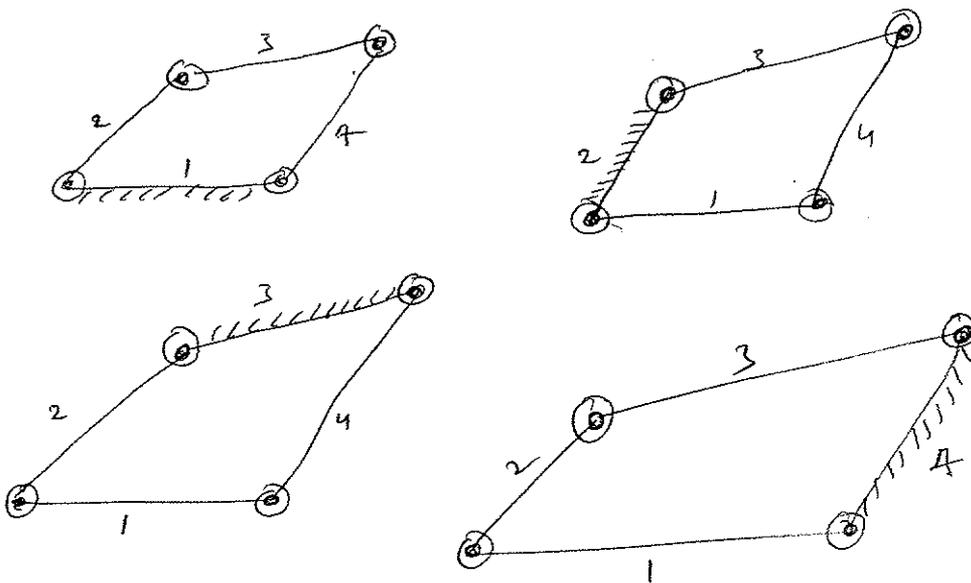
(i) Four Bar Chains :- This is the simplest kinematic chain. It consists of four rigid links which are connected in the form of a quadrilateral by four pin joints as shown in Fig.



It consists of four turning pairs, link 1 and link 2 forms first turning pair, link 2 and link 3 form second turning pair, link 3 and link 4 forms third turning pair, and link 4 and link 1 forms fourth turning pair.

A link that makes complete revolution is known as crank. The fixed link is known as frame of the mechanism. The link opposite to the fixed link is known as connecting rod (or) coupler. The fourth link is known as lever or rocker or an another crank (if it rotates).

If different link of the four bar mechanism are fixed, four different mechanism (known as inversion) as shown in fig.



In case of four-bar mechanism, the following points must be remembered:

- (i) If the length of one of the links is greater than the sum of the lengths of the other three links, four-bar mechanism is not possible.
- (ii) The four links may be of different lengths. But according to Grashof's Law for a four-bar mechanism, the sum of the lengths of the shortest and longest link should not be greater than the sum of lengths of the remaining two links for continuous relative motion b/w the two links.
- (iii) One of the link (shortest link) should make a complete revolution relative to the other three links. The mechanism in which no link makes a complete revolution is not useful.

① INVERSION OF FOUR bar chain :-

Though there are many inversion of the Four bar chain, yet the following are important from the subject point of view.

(a) Beam Engine [Crank and Lever mechanism] :-

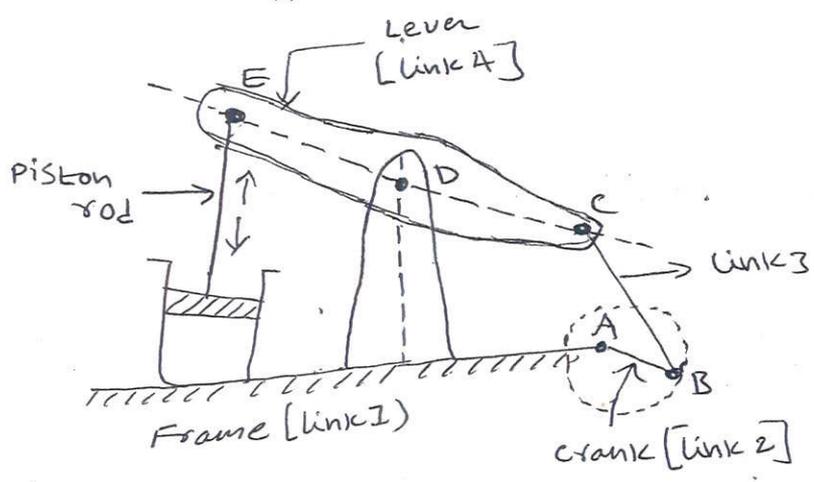


Fig:- Beam Engine

A beam Engine also known as crank and Lever mechanism which consists of four links, shown in above Fig. In this mechanism, when the crank rotates about the fixed center A, the lever oscillates about a fixed center D. The end E of the lever CDE is connected to a piston rod which is reciprocated due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

② Coupling rod of Locomotive [Double crank mechanism] :-

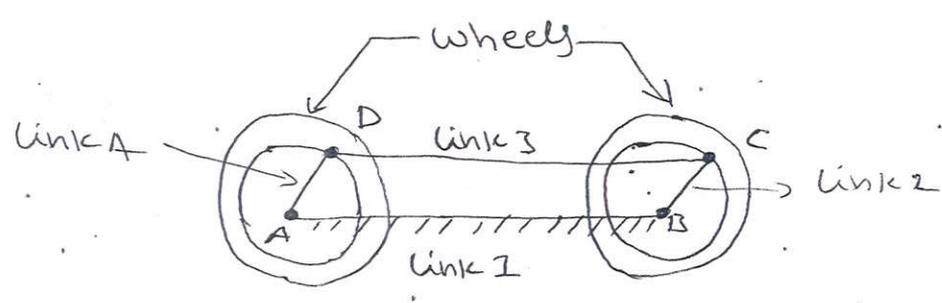


Fig:- Double Crank mechanism

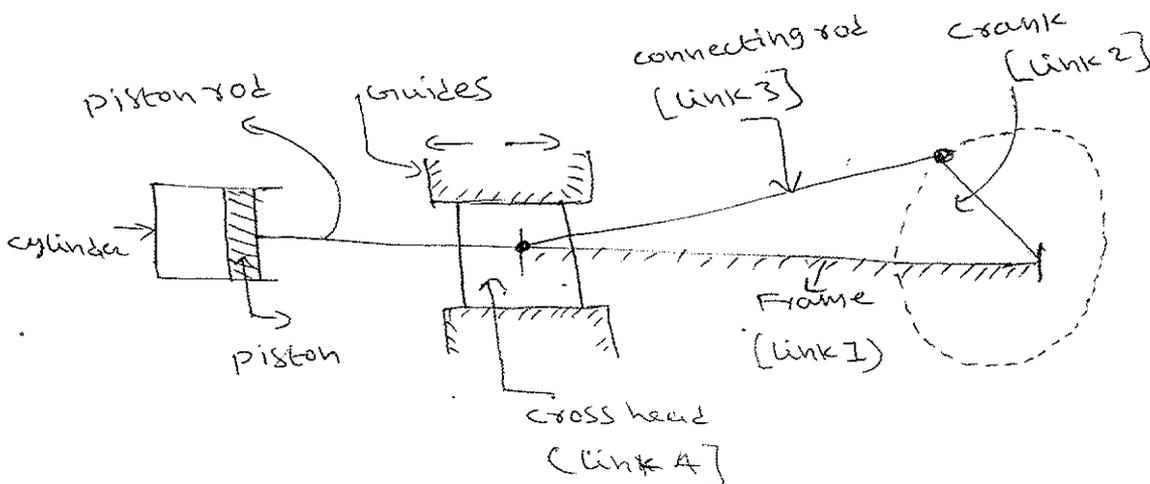
The mechanism of Double Crank mechanism which is consist of four links, is shown in above Fig.

In this mechanism, The links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

### SINGLE SLIDER CRANK CHAIN :-

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pair. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion.

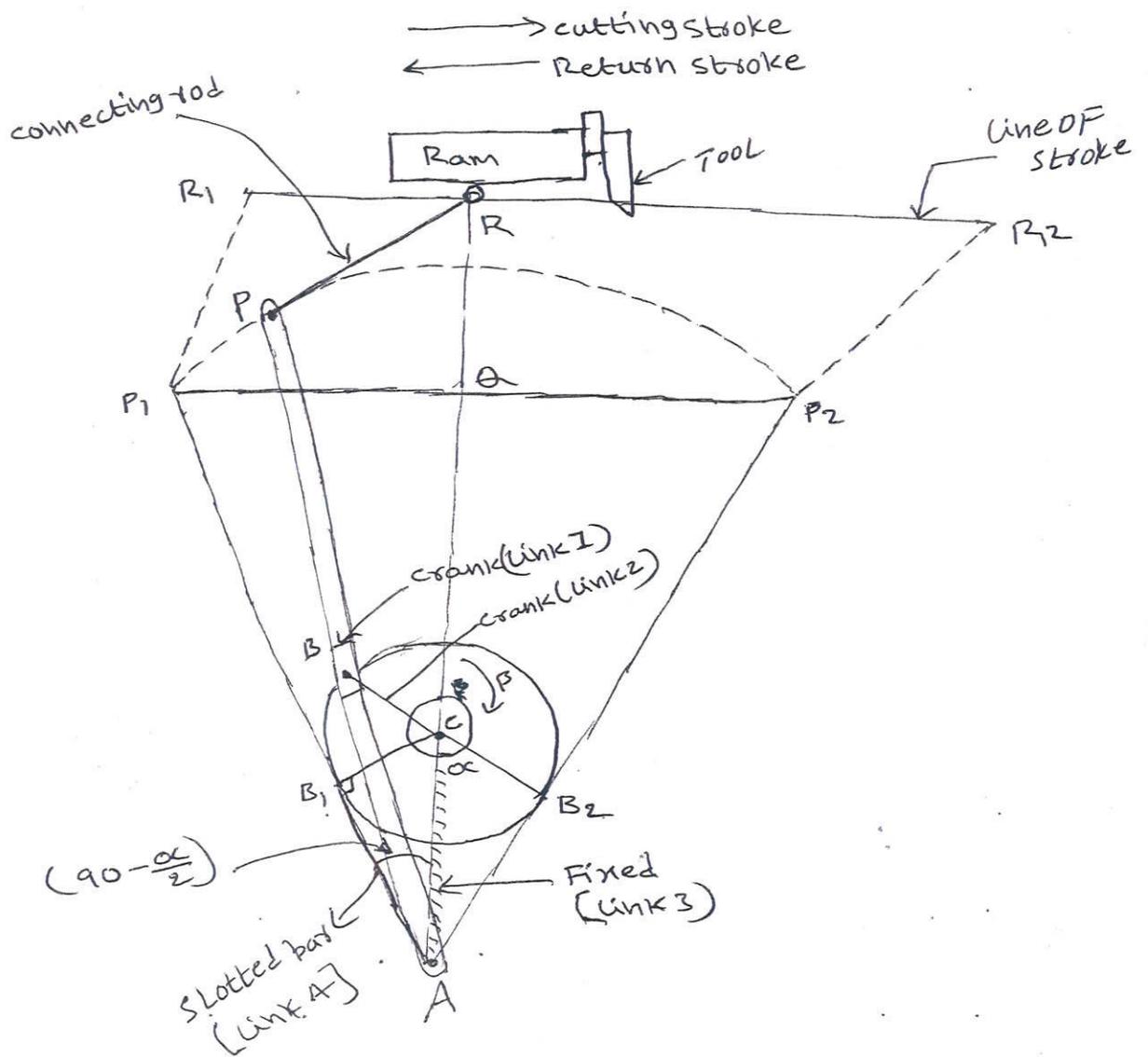
In single-slider crank chain as shown following Fig. The link 1 and 2, link 2 and 3, and link 3 and 4 forms three turning pair while the link 4 and 1 forms a sliding pair.



The link 1 corresponds to the Frame of the engine, which is Fixed.  
 The link 2 corresponds to the crank; Link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-heads reciprocates in the guides and thus the piston reciprocates in the cylinder.

\* crank and slotted Lever quick Return motion mechanism :-

This mechanism is mostly used in Shaping machine, slotting machines and in rotary internal combustion engines.

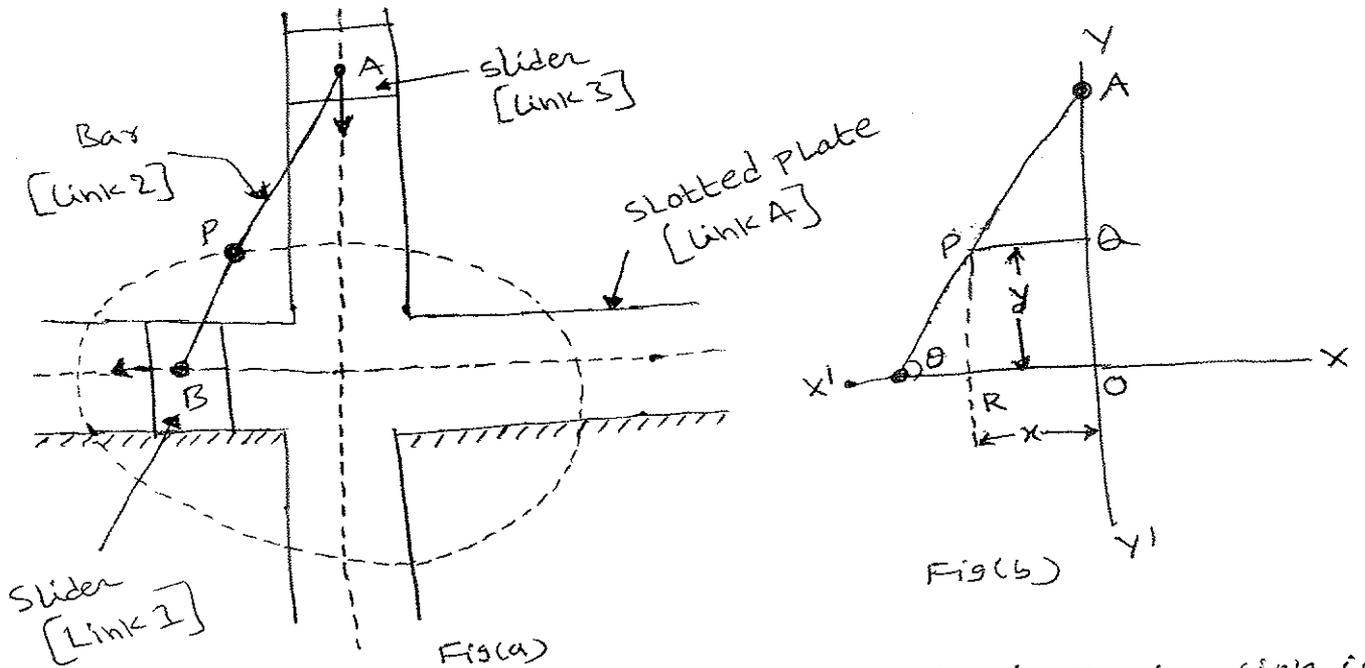


## \* Double slider crank chain :-

A kinetic chain which consists of two turning pairs and two sliding pairs is known as double slider crank chain, as shown in following fig. we see that the link 2 and 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

## \* Inversion of Double slider crank chain :-

### (1) ELLIPTICAL Trammel :-



It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate [Link 4] as shown in fig. The link 1 and link 3, are known as sliders and forms sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with link 1 and 3.

When the link 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4 shown in fig.

Let us take  $OX$  and  $OY$  as horizontal and vertical axes and let the link  $BA$  be inclined at an angle  $\theta$  with the horizontal, as shown in Fig(b). Now the co-ordinates of the point  $P$  on the link  $BA$  will be

$$x = PA = AP \cos \theta \quad \text{and} \quad y = PR = BP \sin \theta$$

$$\frac{x}{AP} = \cos \theta; \quad \text{and} \quad \frac{y}{BP} = \sin \theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2 \theta + \sin^2 \theta = 1$$

This is the equation of an ellipse. Hence the path traced by point  $P$  is an ellipse whose semi-major axis is  $AP$  and semi-minor axis is  $BP$ .

Note:- if ' $P$ ' is the mid-point of link  $BA$ , then  $AP = BP$ . The above equation can be written as

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1 \quad (\text{or}) \quad x^2 + y^2 = (AP)^2$$

This is the equation of a circle whose radius is  $AP$ . Hence if ' $P$ ' is the mid-point of link  $BA$ , it will trace a circle.

## Problem

- ① A crank and slotted lever mechanism used in a shaper has a centre distance of 300mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120mm. Find the ratio of the time of cutting to the time of return stroke

Sol:- Given:  $AC = 300\text{mm}$   
 $CB_1 = 120\text{mm}$

The extreme position of the crank are shown in Fig. we know that

$$\begin{aligned}\sin \angle CAB_1 &= \sin \left[ 90^\circ - \frac{\alpha}{2} \right] \\ &= \frac{CB_1}{AC} = \frac{120}{300} = 0.4\end{aligned}$$

$$\begin{aligned}\angle CAB_1 &= 90 - \frac{\alpha}{2} \\ &= \sin^{-1} 0.4 = 23.6^\circ\end{aligned}$$

$$\frac{\alpha}{2} = 90^\circ - 23.6^\circ = 66.4^\circ$$

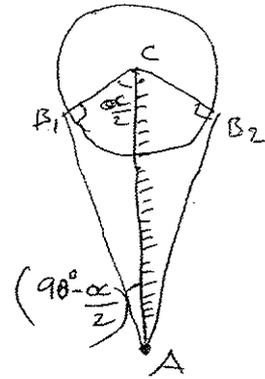
$$\alpha = 2 \times 66.4^\circ$$

$$\alpha = 132.8^\circ$$

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of Return stroke}} = \frac{360 - \alpha}{\alpha} = \frac{360^\circ - 132.8^\circ}{132.8^\circ}$$

$$= 1.72 \text{ Ans.}$$



Q. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240mm and the length of the driving crank is 120mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke.

If the length of the slotted bar is 450mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Sol: Given: -  $AC = 240\text{mm}$   
 $CB_1 = 120\text{mm}$   
 $AP_1 = 450\text{mm}$

Inclination of slotted bar with the vertical :-

Let  $\angle CAB_1 =$  inclination of the slotted bar with the vertical

The extreme position of the crank are shown in Fig. we know that

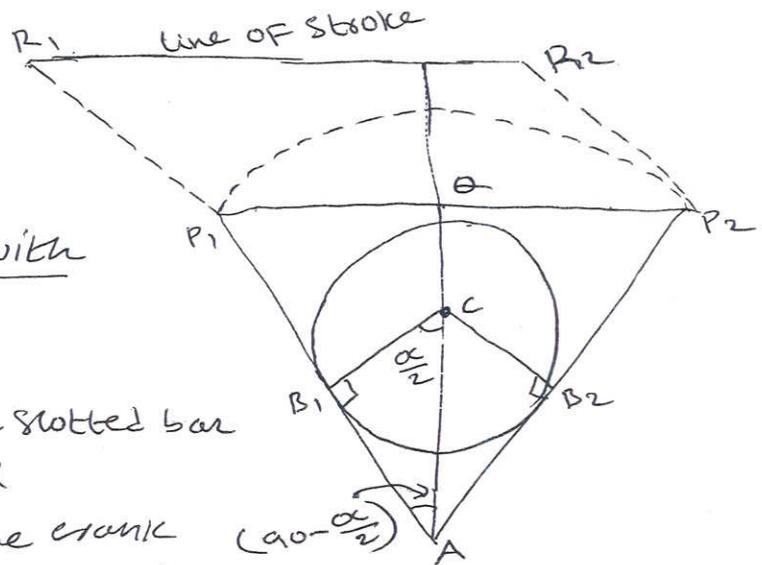
$$\begin{aligned} \sin \angle CAB_1 &= \sin (90^\circ - \alpha/2) \\ &= \frac{B_1C}{AC} = \frac{120}{240} = 0.5 \end{aligned}$$

$$\begin{aligned} \therefore \angle CAB_1 &= 90^\circ - \alpha/2 \\ &= \sin^{-1}(0.5) = 30^\circ \text{ Ans.} \end{aligned}$$

The ratio of cutting stroke to the return stroke :-

we know that

$$\begin{aligned} 90^\circ - \alpha/2 &= 30^\circ \\ \frac{\alpha}{2} &= 90^\circ - 30^\circ = 60^\circ \\ \alpha &= 2 \times 60^\circ = 120^\circ \end{aligned}$$



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360^\circ - \alpha}{\alpha} = \frac{360^\circ - 120^\circ}{120^\circ} = 2 \text{ Avg.}$$

Length of the stroke :

we know that length of the stroke

$$R_1R_2 = P_1P_2 = 2PA = 2AP_1 \sin(90^\circ - \alpha/2)$$

$$= 2 * 450 \sin(90^\circ - 60^\circ)$$

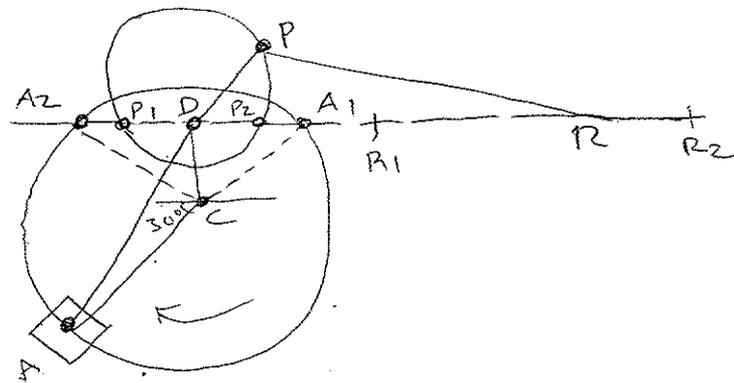
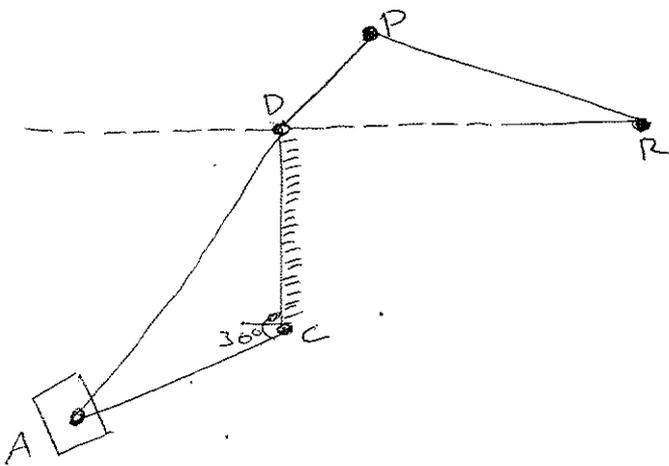
$$= 900 * 0.5$$

$$R_1R_2 = 450 \text{ mm}$$

                    \*

3) In a Whitworth quick return motion mechanism, as shown in Fig. The distance between the fixed centre is 50mm and the length of the driving crank is 75mm. The length of the slotted lever is 150mm, and the length of connecting rod is 135mm. Find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

Sol:- Given:  $CD = 50 \text{ mm}$ ;  $CA = 75 \text{ mm}$ ,  $PA = 150 \text{ mm}$ ;  $PR = 135 \text{ mm}$ ,



The extreme position of the driving crank shown in Fig. From the geometry of the figure,

$$\cos \beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667 \quad \dots \dots \dots (\because CA_2 = CA)$$

$$\beta/2 = 48.2^\circ \quad (\text{or}) \quad \beta = 96.4^\circ$$

Ratio of the time of cutting stroke to the time of return stroke :-

we know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360 - \beta}{\beta} = \frac{360 - 96.4}{96.4} = 2.735 \text{ Ans.}$$

Length of effective stroke :-

In order to find the length of effective stroke (i.e.  $R_1R_2$ ), draw the space diagram of the mechanism to some suitable scale, as shown in Fig. Mark  $P_1R_1 = P_2R_2 = PR$ . Therefore by measurement we find that,

$$\text{Length of effective stroke} = R_1R_2 = 87.5 \text{ mm Ans.}$$